Removal of Undesired Wavefields Related to the Casing of a Microwave Scanner

Peter M. van den Berg and Jacob T. Fokkema

***Abstract*—This paper deals with the scattered field data measured by a closed microwave scanner and discusses the removal of the effects of the metal casing as a preprocessing step before actual inversion of the data can take place. It is shown that Rayleigh's reciprocity theorem furnishes a relation between the scattered field data obtained in the closed microwave scanner and the scattered field data as if they were obtained by an open scanner without the metal casing. This relation leads to a system of linear equations, from which the desired scattered field data with respect to the equivalent open scanner are obtained.**

***Index Terms*—Inverse scattering, microwave scanner, reciprocity.**

I. Introduction

FOR biomedical applications, a number of quantitative microwave imaging algorithms have been discussed, where the object under consideration is assumed to be embedded in an infinite homogeneous medium. Inverse methods, to image the objects quantitatively, are very expensive in terms of the cost of the numerical operations. Recently, for a homogeneous background, some effective inversion procedures [1] have been developed, where, due to the convolutional structure of the Green functions, the underlying algorithm is very efficient as far as computational operations are concerned. For the case of an object located in a homogeneous environment of infinite extent, the scattered field response is conveniently expressed in contrast-source type of integral operators. In view of the convolution structure of these operators, they are efficiently computed with fast Fourier transform (FFT) routines. This feature facilitates the inversion of complex problems.

In most practical situations, the embedding cannot be assumed to be homogeneous with infinite extent. For example, at the Centre National de la Recherche Scientifique (CNRS)/Supélec, Paris, France [2], [3], a cylindrical 434-MHz closed microwave scanner has been developed. This system consists of a water-filled circular cavity with metal casing in which a number of transmitting/receiver dipoles are regularly spaced on a circle of slightly smaller diameter. At the cross section of interest, it is assumed that both the body and sources are cylindrical along the scanner axis so that one suffices with a two-dimensional (2-D) TM scalar field problem, where the fundamental field quantity is the electric-field component parallel to the scanner axis. In modeling the field inside this scanner, Geffrin [2] has taken into account the effects of the metallic casing by computing the Green function of a metallic circular cavity. As far as the computational burden in comparison to the problem with a free-space Green function (of the homogeneous embedding of infinite extent) is concerned, the convolution structure has been lost and the necessary computation time and computer storage for the modeling of the fields may increase dramatically. To avoid this problem, Tijhuis and Franchois [4] have modified their inversion algorithm such that, in each iteration, a forward solver with a free-space Green function can be used. This is achieved by applying in each iteration a proper embedding approach, where the influence of the metal casing is taking into account by changing the sources of the transmitting field.

In order to be able to use inversion algorithms, developed for objects in homogeneous embedding, without any change, we present a preprocessing procedure in which the scattered field data of the closed scanner are replaced by the scattered field data from an open scanner. The sources that generate the wave fields are unchanged. The removal of undesired wavefields related to the metal casing has to be effected without changing any relevant scattered field information of the object to be probed. In relation to a similar marine seismic problem, Fokkema and Van den Berg [5] and van Borselen *et al.* [6] have shown that Rayleigh's reciprocity theorem furnishes the tool for this removal. In such a theorem, the interaction of two nonidentical states is considered. One state is identified with the actual situation, while the other is the desired one; in our case, the same scanner, but without the metal casing. This procedure does not require any information about the object to be probed, neither structural nor material. The removal procedure finally boils down to an inversion of a matrix. It is anticipated that the computational cost of this matrix is relatively quite cheap with respect to the total cost of the inversion algorithm to image the object quantitatively.

II. 2-D TM Field Problem

We use x as the position vector in $\mathbf{R}^2$. We consider harmonic waves with radial frequency ω and use the complex field representation with the time factor $\exp(-st)$, where $s \rightarrow i\omega$ is the complex Laplace parameter, with $\mathrm{Re}(s) > 0$, and i is the imaginary unit. Let $D_{\rm sct}$ denote the domain of the scattering object(s). The objects are nonmagnetic and the sources are of an electric type. Assume that $D_{\rm sct}$ is irradiated by an incident field $u^{\rm inc}$, then the total field $u = u(x)$ at position x satisfies the modified Helmholtz equation

$$(\nabla \cdot \nabla)u - \gamma^2 u = -q \tag{1}$$

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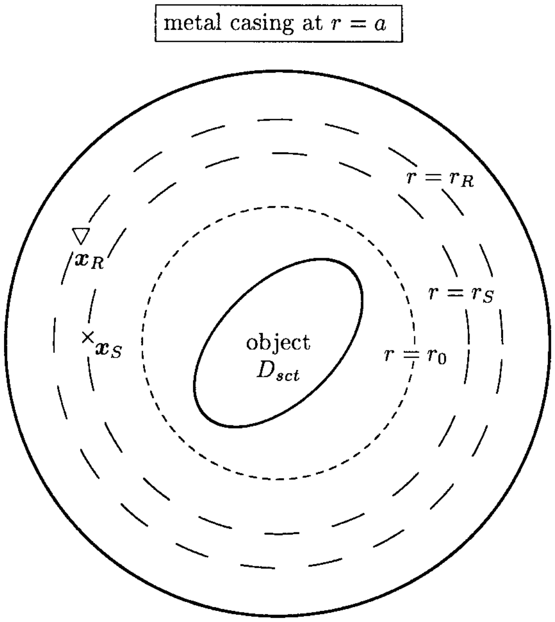

Fig. 1. Closed microwave scanner configuration with transmitter at x_S (denoted by the symbol $\times$) and receiver at x_R (denoted by the symbol ∇).

where q denotes the source distribution. Further, the propagation coefficient $\gamma = \gamma(x)$ is given by

$$\gamma = s/c, \tag{2}$$

where $c = c(x)$ denotes the complex wave speed. In case of the presence of a metal casing in the microwave scanner, the field satisfies the boundary condition that $u = 0$ at the metal casing. For an open scanner, the field should be regular at infinity, satisfying the radiation conditions.

In view of the circular structure of the acquisition and the configuration of the microwave scanners, we introduce a polar coordinate system $x = (r, \varphi)$. In this polar coordinate system, the source position is given by $x_S = (r_S, \varphi_S)$, while the receiver position is given by $x_R = (r_R, \varphi_R)$ (see Figs. 1 and 2). In each circular section, where the medium is homogeneous and no sources are present, for all $\phi \in [0, 2\pi]$, two independent types of wavefield constituents exist, viz.

$$u_m = I_m(\gamma r)\exp(\pm im\varphi), \qquad -\infty < m < \infty \tag{3}$$

which is regular at $r = 0$, and

$$u_m = K_m(\gamma r)\exp(\pm im\varphi), \qquad -\infty < m < \infty \tag{4}$$

which is regular at $r \to \infty$. Both type of field constituents are solutions of the homogeneous wave equation (1). The functions I_m and K_m are modified Bessel functions of order m and of the first and second kinds, respectively. The field constituents are used extensively in the relation between the fields occurring in the closed and open microwave scanners. Before discussing the fields in these two cases in more detail, we first formulate Rayleigh's reciprocity relation.

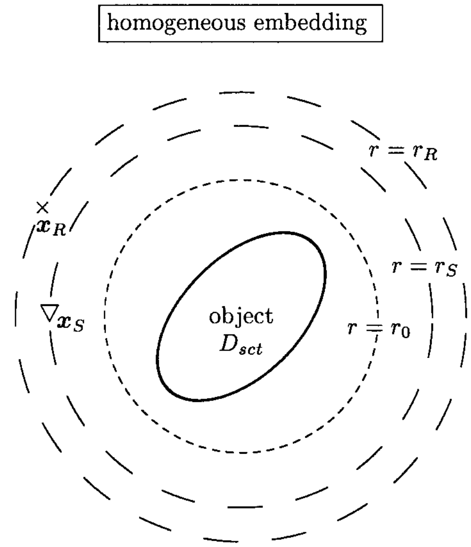

Fig. 2. Open microwave scanner configuration with transmitter at x_R (denoted by the symbol $\times$) and receiver at x_S denoted by the symbol ∇).

III. Reciprocity Relation Between two Different States

In a reciprocity theorem, we consider a time-invariant bounded domain D in space in which two field states occur. The two states will be distinguished by the superscripts A and B, respectively. The boundary surface of D is denoted by ∂D; the normal vector ν on ∂D is directed away from D. State A is characterized by the source distribution q^A generating a wave field u^A. Similarly, state B is characterized by the source distribution q^B and wave field u^B. In both cases, the interior medium properties are the same. In the domain D under consideration, the wave equations for states A and B are

$$(\nabla \cdot \nabla)u^A - \gamma^2 u^A = -\, q^A \tag{5}$$
$$(\nabla \cdot \nabla)u^B - \gamma^2 u^B = -\, q^B \tag{6}$$

respectively. In the reciprocity relation, the interaction quantity between the two states is

$$\nabla \cdot (u^A \nabla u^B - u^B \nabla u^A) = u^A(\nabla \cdot \nabla)u^B - u^B(\nabla \cdot \nabla)u^A. \tag{7}$$

Using wave equations (5) and (6), we arrive at the local reciprocity relation

$$\nabla \cdot (u^A \nabla u^B - u^B \nabla u^A) = q^A u^B - q^B u^A. \tag{8}$$

Using Gauss' theorem, we obtain the global form

$$\int_{\partial D} \left(u^A \partial_\nu u^B - u^B \partial_\nu u^A\right) \mathrm{ds} = \int_{D} \left(q^A u^B - q^B u^A\right) \mathrm{dA} \tag{9}$$

where ∂_ν denotes the spatial derivative in the direction of the normal ν to ∂D. We note that (9) is Rayleigh's reciprocity theorem for the domain D when the internal medium properties of the two states are the same (Rayleigh [7]; he denoted it as Helmholtz' theorem).

We subsequently define the two field states in more detail. In particular, state A characterizes the field state inside a closed microwave scanner and state B characterizes the field state in an open scanner.

A. State A: The Wave Field Inside a Closed Scanner

Define state A as the field in the closed scanner, where this field is generated by a source at $x = x_S$ and source strength $2\pi Q$, i.e.,s

$$q^A = 2\pi Q\,\delta(x - x_S). \tag{10}$$

The incident field in the scanner is defined as the field in absence of the object $D_{\rm sct}$. This incident field consists of a direct field from the source and a reflected contribution due to the presence of the metal casing at $r = a$ (see Fig. 1). Since this incident field is regular at $r = 0$, we write this field in the form

$$u^{A,\rm inc}(x, x_S) = QK_0\big(\gamma|x - x_S|\big) + Q\sum_{m=-\infty}^{\infty}\exp\big[im(\varphi - \varphi_S)\big]R_m^{\rm inc}I_m(\gamma r) \tag{11}$$

where the first term is the direct field from the source at $x = x_S$ and the second term (with reflection factor $R_m^{\rm inc}$) is the contribution due to the presence of the metal casing at $r = a$. Note that, in view of the addition theorem of Bessel functions, we may write

$$K_0\big(\gamma|x - x_S|\big) = \sum_{m=-\infty}^{\infty}\exp\big[im(\varphi - \varphi_S)\big]I_m(\gamma r_<)K_m(\gamma r_>) \tag{12}$$

where $r_< = \min\{r, r_S\}$ and $r_> = \max\{r, r_S\}$. In view of the boundary condition that $u^{A,\rm inc} = 0$ at the casing (for $r = a$), the reflection factor $R_m^{\rm inc}$ is obtained as

$$R_m^{\rm inc} = -\frac{I_m(\gamma r_S)K_m(\gamma a)}{I_m(\gamma a)}. \tag{13}$$

From the experiments of the fluid-filled scanner without the presence of object $D_{\rm sct}$, the source strength is obtained from the average value of all measurements $u^{A,\rm inc}(x_R, x_S)$ as

$$\frac{1}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi} u^{A,\rm inc}(x_R, x_S){\rm d}\varphi_R{\rm d}\varphi_S = Q\big[I_0(\gamma r_<)K_0(\gamma r_>) + R_0^{\rm inc}I_0(\gamma r_R)\big] \tag{14}$$

where $r_< = \min\{r_R, r_S\}$ and $r_> = \max\{r_R, r_S\}$. From this relation, the quantity Q can be determined robustly. With this value for Q, the incident field in the fluid-filled scanner with no object is completely described.

The scattered field of state A is defined as the field caused by the presence of the object $D_{\rm sct}$, hence,

$$u^{A,\rm sct} = u^A - u^{A,\rm inc}. \tag{15}$$

Let $r = r_0$ describe a circle that completely encloses the object $D_{\rm sct}$. We assume that r_0 is always less than both r_R and r_S. In the homogeneous domain $r_0 \le r < a$ outside the object $D_{\rm sct}$, this scattered field can then be decomposed in the wave constituents of both the type of (4) and (3). We write this scattered field as

$$u^{A,\rm sct}(x, x_S) = \sum_{m=-\infty}^{\infty}\exp(im\varphi)\,u_m^{A,\rm sct}(\varphi_S) \times\frac{K_m(\gamma r) + R_m^{\rm sct}I_m(\gamma r)}{K_m(\gamma r_R) + R_m^{\rm sct}I_m(\gamma r_R)}. \tag{16}$$

The factor $R_m^{\rm sct}$ represent the reflection factor due to the metal casing. In view of the boundary condition that $u^A = 0$ and $u^{A,\rm inc} = 0$ at the metal casing for $r = a$, the scattered field $u^{A,\rm sct} = 0$ vanishes there as well, hence, the reflection factor $R_m^{\rm sct}$ is obtained as

$$R_m^{\rm sct} = -\frac{K_m(\gamma a)}{I_m(\gamma a)}. \tag{17}$$

Note further that the denominator at the right-hand side of (16) is added for convenience to arrive at the situation that, at the receiver level $r = r_R$, we have

$$u^{A,\rm sct}(x_R, x_S) = \sum_{m=-\infty}^{\infty}\exp(im\varphi_R)\,u_m^{A,\rm sct}(\varphi_S). \tag{18}$$

Hence, the coefficients $u_m^{A,\rm sct}(\varphi_S)$ are the Fourier coefficients of the scattered field data as far as the transform with respect to the receiver coordinates is concerned. Using the inverse Fourier transform, we observe that the quantity $u_m^{A,\rm sct}(\varphi_S)$ follows from the measured data $u^{A,\rm sct}(x_R, x_S)$ as

$$u_m^{A,\rm sct}(\varphi_S) = \frac{1}{2\pi}\int_0^{2\pi}\exp(-im\varphi_R)\,u^{A,\rm sct}(x_R, x_S)\,{\rm d}\varphi_R. \tag{19}$$

B. State B: The Wave Field Inside an Open Scanner

Define state B as the desired field in the equivalent open scanner, where this field is generated by a source at the receiver location $x = x^R$ [see Fig. (2)] and source strength $2\pi Q$, i.e.,

$$q^B = 2\pi Q\,\delta(x - x_R). \tag{20}$$

In state B, the source and receiver locations are interchanged. This means that the sources are at $r = r_R$ and the receivers are at r_S. Now, the incident field consists of a direct field from the source only so that this field is simply written as

$$u^{B,\rm inc}(x, x_R) = Q\,K_0(\gamma|x - x_R|) \tag{21}$$

with

$$K_0(\gamma|x - x_R|) = \sum_{m=-\infty}^{\infty}\exp\big[im(\varphi_R - \varphi)\big] \times I_m(\gamma r_<)K_m(\gamma r_>) \tag{22}$$

where $r_< = \min\{r, r_R\}$ and $r_> = \max\{r, r_R\}$.

Let the scattered field be defined as

$$u^{B,\rm sct} = u^B - u^{B,\rm inc}. \tag{23}$$

Again let $r = r_0$ describe a circle that completely encloses the object $D_{\rm sct}$. In the homogeneous domain $r_0 \le r < \infty$ outside the object $D_{\rm sct}$, this scattered field can then be written as

$$u^{B,\rm sct}(x, x_R) = \sum_{m=-\infty}^{\infty} \exp(-im\varphi) u_m^{B,\rm sct}(\varphi_R) \frac{K_m(\gamma r)}{K_m(\gamma r_S)}. \tag{24}$$

Note further that the denominator at the right-hand side of (24) is added for convenience to arrive at the situation that, at the level $r = r_S$, we have

$$u^{B,\rm sct}(x_S, x_R) = \sum_{m=-\infty}^{\infty} \exp(-im\varphi_S)\, u_m^{B,\rm sct}(\varphi_R). \tag{25}$$

Hence, the coefficients $u_m^{B,\rm sct}(\varphi_R)$ are the Fourier coefficients of the scattered field data as far as the transform with respect to the receiver coordinates (in this case, φ_S) is concerned. Using the inverse Fourier transform, we observe that the quantities $u_m^{B,\rm sct}(\varphi_R)$ may be obtained from the data $u^{B,\rm sct}(x_S, x_R)$ as

$$u_m^{B,\rm sct}(\varphi_R) = \frac{1}{2\pi} \int_0^{2\pi} \exp(im\varphi_S)\, u^{B,\rm sct}(x_S, x_R)\, \mathrm{d}\varphi_S. \tag{26}$$

IV. Removal Procedure

To relate the two states described in the previous section to each other, we apply the global reciprocity relation of (9) to the source-free domain $0 \le r < r_0$ so that we obtain the relation

$$\int_0^{2\pi} (u^A \partial_r u^B - u^B \partial_r u^A)\, r\, \mathrm{d}\varphi = 0, \qquad r = r_0 \tag{27}$$

and substitute in this relation the expressions for $u^A = u^{A,\rm inc} + u^{A,\rm sct}$ and $u^B = u^{B,\rm inc} + u^{B,\rm sct}$ to arrive at the "*propagation invariant*"

$$\sum_{m=-\infty}^{\infty} \langle U_m^A, U_m^B \rangle = 0 \tag{28}$$

where

$$\langle U_m^A, U_m^B \rangle = U_m^A \partial_{r_0} U_m^B - U_m^B \partial_{r_0} U_m^A \tag{29}$$

and

$$U_m^A = U_m^{A,\rm inc} + U_m^{A,\rm sct} \tag{30}$$

$$U_m^{A,\rm inc} = Q\, I_m(\gamma r_0) \big[K_m(\gamma r_S) + R_m^{\rm inc}\big] \exp(-im\varphi_S) \tag{31}$$

$$U_m^{A,\rm sct} = u_m^{A,\rm sct}(\varphi_S) \frac{K_m(\gamma r_0) + R_m^{\rm sct} I_m(\gamma r_0)}{K_m(\gamma r_R) + R_m^{\rm sct} I_m(\gamma r_R)} \tag{32}$$

while

$$U_m^B = U_m^{B,\rm inc} + U_m^{B,\rm sct} \tag{33}$$

$$U_m^{B,\rm inc} = Q I_m(\gamma r_0) K_m(\gamma r_R) \exp(im\varphi_R) \tag{34}$$

$$U_m^{B,\rm sct} = u_m^{B,\rm sct}(\varphi_R) \frac{K_m(\gamma r_0)}{K_m(\gamma r_S)}. \tag{35}$$

Using the relations

$$\big\langle K_m(\gamma r_0), K_m(\gamma r_0) \big\rangle = 0 \tag{36}$$

$$\big\langle I_m(\gamma r_0), I_m(\gamma r_0) \big\rangle = 0 \tag{37}$$

which directly follow from (29), and

$$\big\langle K_m(\gamma r_0), I_m(\gamma r_0) \big\rangle = -\big\langle I_m(\gamma r_0), K_m(\gamma r_0) \big\rangle = \frac{1}{r_0} \tag{38}$$

which follows from the Wronskian for modified Bessel functions ([8, form. 9.6.15]), we obtain

$$\sum_{m=-\infty}^{\infty} \Bigg\{ Q \frac{K_m(\gamma r_S) + R_m^{\rm inc}}{K_m(\gamma r_S)} \exp(-im\varphi_S) u_m^{B,\rm sct}(\varphi_R) + \frac{u_m^{A,\rm sct}(\varphi_S)\, R_m^{\rm sct}}{K_m(\gamma r_R) + R_m^{\rm sct} I_m(\gamma r_R)} \frac{1}{K_m(\gamma r_S)} u_m^{B,\rm sct}(\varphi_R) - Q \frac{u_m^{A,\rm sct}(\varphi_S)\, K_m(\gamma r_R)}{K_m(\gamma r_R) + R_m^{\rm sct} I_m(\gamma r_R)} \exp(im\varphi_R) \Bigg\} = 0. \tag{39}$$

We now apply the inverse Fourier transform with respect to $\varphi_S, 1/2\pi \int_0^{2\pi} \mathrm{d}\varphi_S \exp(ik\varphi_S)$, and subsequently apply the inverse Fourier transform with respect to $\varphi_R, 1/2\pi \int_0^{2\pi} \mathrm{d}\varphi_R \exp(-il\varphi_R)$ to all terms of (39). With the definitions

$$u_{k,m}^{A,\rm sct} = \frac{1}{2\pi} \int_0^{2\pi} \exp(ik\varphi_S) u_m^{A,\rm sct}(\varphi_S) \mathrm{d}\varphi_S \tag{40}$$

$$u_{m,l}^{B,\rm sct} = \frac{1}{2\pi} \int_0^{2\pi} \exp(-il\varphi_R) u_m^{B,\rm sct}(\varphi_R) \mathrm{d}\varphi_R \tag{41}$$

we arrive at

$$Q \frac{K_k(\gamma r_S) + R_k^{\rm inc}}{K_k(\gamma r_S)} u_{k,l}^{B,\rm sct} + \sum_{m=-\infty}^{\infty} \frac{u_{k,m}^{A,\rm sct}\, R_m^{\rm sct}}{K_m(\gamma r_R) + R_m^{\rm sct} I_m(\gamma r_R)} \frac{1}{K_m(\gamma r_S)} u_{m,l}^{B,\rm sct} - Q \frac{u_{k,l}^{A,\rm sct}\, K_l(\gamma r_R)}{K_l(\gamma r_R) + R_l^{\rm sct} I_l(\gamma r_R)} = 0, \qquad -\infty < k < \infty \tag{42}$$

which is, for each l, an infinite system of equations, from which $u_{m,l}^{B,\rm sct}$ has to be solved. This system of equations is written as

$$x_{k,l} + \sum_{m=-\infty}^{\infty} A_{k,m}\, x_{m,l} = b_{k,l}, \qquad -\infty < k < \infty \tag{43}$$

with the system matrix

$$A_{k,m} = \frac{1}{Q} \frac{K_k(\gamma r_S)}{K_k(\gamma r_S) + R_k^{\rm inc}} u_{k,m}^{A,\rm sct} \times \frac{R_m^{\rm sct}}{K_m(\gamma r_S) \big[K_m(\gamma r_R) + R_m^{\rm sct} I_m(\gamma r_R)\big]} \tag{44}$$

the known vector

$$b_{k,l} = \frac{K_k(\gamma r_S)}{K_k(\gamma r_S) + R_k^{\rm inc}} u_{k,l}^{A,\rm sct} \frac{K_l(\gamma r_R)}{K_l(\gamma r_R) + R_l^{\rm sct} I_l(\gamma r_R)} \tag{45}$$

and the unknown vector

$$x_{m,l} = u_{m,l}^{B,\rm sct}. \tag{46}$$

The formal solution of (43) is obtained as

$$x_{k,l} = \sum_{m=-\infty}^{\infty} [I + A]^{-1}_{k,m}\, b_{m,l} \tag{47}$$

where I is the diagonal unit matrix. With this solution, the scattered field in the open microwave scanner is obtained from (25), where the coefficients $u_m^{B,\mathrm{sct}}(\phi_R)$ in accordance to (41) are given by

$$u_m^{B,\mathrm{sct}}(\phi_R) = \sum_{l=-\infty}^{\infty} \exp(il\varphi_R)\, x_{m,l}. \tag{48}$$

When we substitute this result into (25), we obtain the scattered field data $u^{B,\mathrm{sct}}(x_S, x_R)$. Using the standard source/receiver reciprocity, we obtain the desired scattered field data of the equivalent open microwave scanner as

$$\begin{aligned} u^{B,\mathrm{sct}}(x_R, x_S) &= u^{B,\mathrm{sct}}(x_S, x_R) \\ &= \sum_{m=-\infty}^{\infty} \exp(-im\varphi_S) u_m^{B,\mathrm{sct}}(\varphi_R). \end{aligned} \tag{49}$$

The scattered field data for the equivalent open scanner can be used as input for the inversion algorithms dealing with objects in a homogeneous embedding of infinite extent.

We remark that inversion schemes based on local type of field solutions, like finite elements or finite differences, do not benefit from the present procedure. In order to bound the domain of computation, there may be a strong preference to operate with the closed scanner.

In the case that we are dealing with experimental scattered field data obtained in an open microwave scanner, and one prefers an equivalent closed scanner so that local type of field solutions may be applied efficiently, then we need a preprocessing step to replace the open scanner by an equivalent closed scanner. This is discussed in Section V.

V. Replacement of an Open Microwave Scanner by a Closed One

We start with the remark that (42) yields the relation between the open and closed scanners. However, now we envisage it, for each k, as an infinite system of equations, from which $u_{k,m}^{A,\mathrm{sct}}$ has to be solved. This follows immediately by rearranging this equation as follows:

$$\begin{aligned} &- Q\frac{K_l(\gamma r_R)}{K_l(\gamma r_R) + R_l^{\mathrm{sct}} I_l(\gamma r_R)} u_{k,l}^{A,\mathrm{sct}} \\ &+ \sum_{m=-\infty}^{\infty} \frac{u_{m,l}^{B,\mathrm{sct}}\, R_m^{\mathrm{sct}}}{K_m(\gamma r_R) + R_m^{\mathrm{sct}} I_m(\gamma r_R)} \frac{1}{K_m(\gamma r_S)}\, u_{k,m}^{A,\mathrm{sct}} \\ &+ Q\frac{u_{k,l}^{B,\mathrm{sct}}\left[K_k(\gamma r_S) + R_k^{\mathrm{inc}}\right]}{K_k(\gamma r_S)} = 0, \qquad -\infty < l < \infty. \end{aligned} \tag{50}$$

This system of equations is written as

$$x_{k,l} - \sum_{m=-\infty}^{\infty} x_{k,m}\, A_{m,l} = b_{k,l}, \qquad -\infty < l < \infty \tag{51}$$

with the system matrix

$$\begin{aligned} A_{m,l} = \frac{1}{Q} &\frac{R_m^{\mathrm{sct}}}{K_m(\gamma r_S)\left[K_m(\gamma r_R) + R_m^{\mathrm{sct}} I_m(\gamma r_R)\right]} \\ &\times u_{m,l}^{B,\mathrm{sct}} \frac{K_l(\gamma r_R) + R_l^{\mathrm{sct}} I_l(\gamma r_R)}{K_l(\gamma r_R)} \end{aligned} \tag{52}$$

the known vector

$$b_{k,l} = \frac{K_k(\gamma r_S) + R_k^{\mathrm{inc}}}{K_k(\gamma r_S)}\, u_{k,l}^{B,\mathrm{sct}} \frac{K_l(\gamma r_R) + R_l^{\mathrm{sct}} I_l(\gamma r_R)}{K_l(\gamma r_R)} \tag{53}$$

and the unknown vector

$$x_{k,m} = u_{k,m}^{A,\mathrm{sct}}. \tag{54}$$

The formal solution of (51) is obtained as

$$x_{k,l} = \sum_{m=-\infty}^{\infty} b_{k,m}\, [I - A]^{-1}_{m,l}. \tag{55}$$

With this solution, the scattered field in the closed microwave scanner is obtained from (18), where the coefficients $u_m^{A,\mathrm{sct}}(\phi_S)$ in accordance to (40) are given by

$$u_m^{A,\mathrm{sct}}(\phi_S) = \sum_{k=-\infty}^{\infty} \exp(ik\varphi_S)\, x_{k,m} \tag{56}$$

and, consequently, we obtain the scattered field data $u^{A,\mathrm{sct}}(x_R, x_S)$ as [cf. (18)]

$$u^{A,\mathrm{sct}}(x_R, x_S) = \sum_{m=-\infty}^{\infty} \exp(im\varphi_R)\, u_m^{A,\mathrm{sct}}(\varphi_S). \tag{57}$$

The scattered field data for the equivalent closed scanner can now be used as input for inversion algorithms dealing with the local type of field solutions.

VI. Conclusions

In this paper, we have shown that the scattered field data obtained in a closed microwave scanner and the scattered field data obtained by an equivalent open scanner are related to each other via Rayleigh's reciprocity theorem. When the scattered field data from one type of scanner is known, the scattered field data from the other type of scanner is obtained by carrying out a simple processing step, which involves an inversion of a system of linear equations. In practice, the number of sources that generate the fields and the number of receivers that measure the fields are limited. This means that all the numbers m, l, and k have a bounded range. The Fourier inversion integrals have to be replaced by finite summations as well. The several summations can be carried out efficiently with standard FFT routines.

We finally mention that equivalent problems in acoustics, elastodynamics, and three-dimensional (3-D) electromagnetic may be analyzed in a similar way using Rayleigh's reciprocity theorem for acoustic waves, Betti's reciprocity for elastodynamics, and Lorentz' reciprocity theorem for electromagnetic waves, respectively. A fine overview of these reciprocity theorems with a number of applications can be found in De Hoop's handbook [9].

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